Name: ____________________________

Math 315, Section 2
Exam 1
Instructor: David G. Wright
29-31 January 2009

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1. (20%) Give an example of each of the following or argue that such a request is impossible:

   (a) an unbounded sequence with a convergent subsequence;

   (b) a nested sequence of open intervals whose intersection is empty;

   (c) an unbounded sequence (a_n) and a convergent sequence (b_n) so that $(a_n - b_n)$ is bounded;

   (d) a bounded monotone sequence that has a divergent subsequence.

2. (10%) State and prove the Archimedean Property.

3. (10%) Assume that for any two positive real numbers $a < b$, there exists a rational number r satisfying $a < r < b$. Prove that for any two negative numbers $c < d$, there is a rational number s with $c < s < d$.

4. (10%) Show that the real numbers $\mathbb{R}$ are uncountable.

5. (10%) Prove that the sequence defined by $a_1 = 1$ and $a_{n+1} = 3 - \dfrac{1}{a_n}$ is increasing and $a_n < 3$ for all n. Explain why a_n is convergent and find its limit. Hint: First show $1 \leq a_n < 3$ for all n.

6. (20%) Let $\lim a_n = a$ and $\lim b_n = b$. prove:

   (a) $\lim(a_n + b_n) = a + b$:

   (b) $\lim(a_n b_n) =$ ab.

7. (10%) Show that if $0 < r < 1$, then $\lim r^n = 0$.

8. (10%) Define what it means for a sequence to be Cauchy and prove a Cauchy sequence converges.